Appendix B
The BMA Skew Adjustment Formula

This appendix was prepared with substantial assistance from Andrew Kalotay Associates, Inc.

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A. Introduction

This Appendix B describes in more detail the BMA Skew Adjustment Formula for deriving the Skew Adjusted Volatility associated with a GSE European Callable Security (“ECS”). A particular Skew Adjusted Volatility is used in the BMA ECS Formula described in Appendix A, when computing the Purchase Price and the OAS Associated Quotation of an ECS.

Capitalized terms not otherwise defined herein shall have the meaning set forth in the Guidelines and in Appendix A.

The formula for the Skew Adjusted Volatility utilizes the so–called Blyth–Uglum approximation[1], assuming that swap rates follow a normal rather than a lognormal random process.

B. Definitions

1. **Adjustment Factor.** The term “Adjustment Factor” means the factor that is used to convert the Base Volatility into the Skew Adjusted Volatility. It is given by

$$A = \sqrt{\frac{F}{c}}, \tag{1}$$

   where F is the OAS–Adjusted Forward Par Yield and c is the Coupon Rate as defined in Appendix A.

2. **Base Volatility.** The term “Base Volatility” means the volatility obtained by bilinear interpolation using the Volatility Matrix and the terms of the ECS.

   Let E be the Call Date of the ECS and M the Year-Fractions Between the Call Date and the Maturity Date.

   Given two dates s and t, let $t - s$ denote the number of days from the date s to the date t.

   The Base Volatility is denoted v_{base} and is given by

$$v_{\text{base}} = a_{j,k}V(e_j, M_k) + a_{j+1,k}V(e_{j+1}, M_k) + a_{j,k+1}V(e_j, M_{k+1}) + a_{j+1,k+1}V(e_{j+1}, M_{k+1}),$$

[1] Stephen Blyth and John Uglum, *Rates of Skew*, Risk (1999).

where

$$
\begin{aligned}
a_{j,k} &= \frac{(E_{j+1} - E)(M_{k+1} - M)}{(E_{j+1} - E_j)(M_{k+1} - M_k)} \\
a_{j+1,k} &= \frac{(E - E_j)(M_{k+1} - M)}{(E_{j+1} - E_j)(M_{k+1} - M_k)} \\
a_{j,k+1} &= \frac{E_{j+1} - E)(M - M_k)}{(E_{j+1} - E_j)(M_{k+1} - M_k)} \\
a_{j+1,k+1} &= \frac{(E - E_j)(M - M_k)}{(E_{j+1} - E_j)(M_{k+1} - M_k)},
\end{aligned}
$$

where j and k are chosen so that $E_j \leq E \leq E_{j+1}$ and $M_k \leq M \leq M_{k+1}$. If the date E is after E_a, then use the formulas above with $E_j = E_a$ and $E_{j+1} = E$. If $M > M_b$, then use the formulas above with $M_k = M_b$ and $M_{k+1} = M$.

3. **Skew Adjusted Volatility.** The term "Skew Adjusted Volatility" means the volatility obtained by applying the Blyth–Uglum approximation to the Base Volatility. It is denoted in this Appendix B by v_{adjusted}.

4. **Skew Adjustment Formula** The term "Skew Adjustment Formula" means the formula

$$v_{\text{adjusted}} = A v_{\text{base}}, \tag{2}$$

where v_{adjusted} is the Skew Adjusted Volatility, A is the Adjustment Factor given by (1), and v_{base} is the Base Volatility.

5. **Swap Tenor.** The term "Swap Tenor" means the tenor in years of the swap underlying a swaption.

   The Swap Tenors represented in the Volatility Matrix are denoted by $M_1, \ldots, M_b$.

6. **Swaption Exercise Date.** The term "Swaption Exercise Date" means the date obtained by adding the Time-to-Exercise to the Trade Date.

   The Swaption Exercise Dates obtained by adding the Times-to-Exercise $e_1, \ldots, e_a$ to the Trade Date will be denoted $E_1, \ldots, E_a$.

7. **Time-to-Exercise.** The term "Time-to-Exercise" refers to the time period from the Trade Date to the Swaption Exercise Date. It is specified in either months or years.

   The Times–to–Exercise represented in the Volatility Matrix are denoted by $e_1, \ldots, e_a$.

8. **Trade Date.** The term "Trade Date" has the meaning set forth in Appendix A.

9. **Volatility Matrix.** The term "Volatility Matrix" means a matrix of live at–the–money European swaption volatilities indexed by the Time–to–Exercise and Swap Tenor.

C. Using the Formula The Skew Adjusted Volatility should be computed using the Skew Adjustment Formula and used in the computation of the Option Price in Appendix A.