Appendix A
The BMA European Callable Securities Formula

This appendix was prepared with substantial assistance from Andrew Kalotay Associates, Inc.

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A. Introduction

This Appendix A describes in more specific detail the BMA ECS Formula ("Formula") for calculating the Purchase Price of a particular GSE European Callable Security ("ECS") from its option adjusted spread ("OAS") relative to the Designated Yield Curve. Capitalized terms not otherwise defined herein shall have the meaning set forth in the Guidelines, Appendix B, and Appendix C.

An ECS contains an embedded European option, which allows the Issuer to call the security at par on a particular Coupon Payment Date. The Formula utilizes the so-called Black '76 Formula to price this option. The Formula also relies on OAS-adjusted discounting of the relevant cash flows.

Please note that the Formula is only applicable under the requirements stated in Section E of the Guidelines.

The BMA ECS Formula described in this Appendix A was developed for the sole purpose of facilitating voluntary trading in GSE European Callable Securities. It is proprietary to the Association, which grants you the royalty–free right to use them. Users are cautioned that the mathematical formulas described below are not necessarily suitable for usage with any other types of callable and putable securities, including other European-style callable and putable securities. Users should also be aware that the BMA ECS Formula is not necessarily suitable for usage with yield curves other than the Designated Yield Curve.

B. Definitions

1. **Accrual Year–Fraction.** The term "Accrual Year–Fraction", denoted $y(t_0, T)$, is the Year–Fractions Between Two Dates t_0, which is the Start of the Current Coupon Period, and T, which is the Actual Settlement Date.

2. **Accrued Interest.** The term "Accrued Interest" of an ECS means the amount of interest accrued from the Start of the Current Coupon Period t_0 to the Actual Settlement Date T. The Accrued Interest is denoted by $A(T)$ and is given by

$$A(T) = cy(t_0, T),$$

where c is the Coupon Rate of the ECS and $y(t_0, T)$ is the Accrual Year–Fraction.

3. **Actual Settlement Date.** The term “Actual Settlement Date” means the settlement date of the OAS Trade. It will be denoted throughout this Appendix A by T.

4. **Baseline Discount Factor Curve.** The Baseline Discount Factor Curve refers to the function that assigns to each date t after the Standard Settlement Date T_0 a Baseline Discount Factor $d(T_0, t)$.

5. **Baseline Discount Factor.** The term “Baseline Discount Factor” for a date t is denoted by $d(T_0, t)$. It is obtained from the corresponding Baseline Spot Rate $z(T_0, t)$ using the formula
$$d(T_0, t) = \left(1 + \frac{z(T_0, t)}{200}\right)^{-2y(T_0,t)}, \tag{1}$$
where $y(T_0, t)$ is the Year–Fractions Between the Standard Settlement Date T_0 and the date t.

6. **Baseline Forward Discount Factor.** The term “Baseline Forward Discount Factor” for the coupon period ending on a Coupon Payment Date t_k is denoted by $d(T, t_1)$, if $k = 1$ and $d(t_{k-1}, t_k)$, if $k \geq 1$. It is given by
$$\begin{aligned} d(T, t_1) &= \frac{d(T_0, t_1)}{d(T_0, T)}, \qquad \text{if } k = 1 \\ d(t_{k-1}, t_k) &= \frac{d(T_0, t_k)}{d(T_0, t_{k-1})}, \quad \text{if } k > 1, \end{aligned} \tag{2}$$
where T is the Actual Settlement Date and $d(T_0, s)$ is the Baseline Discount Factor for the date s.

7. **Baseline One–Period Forward Rate.** The term “Baseline One–Period Forward Rate” for the coupon period ending on a Coupon Payment Date t_k is denoted by $f(T, t_1)$, if $k = 1$ and $f(t_{k-1}, t_k)$, if $k > 1$. It is given by
$$\begin{aligned} f(T, t_1) &= \frac{100}{y_1}\left(\frac{1}{d(T, t_1)} - 1\right) &&, \text{ if } k = 1, \\ f(t_{k-1}, t_k) &= 200\left(\frac{1}{d(t_{k-1}, t_k)} - 1\right) &&, \text{ if } 1 < k < N, \\ f(t_{N-1}, t_N) &= \frac{100}{y_N}\left(\frac{1}{d(t_{N-1}, t_N)} - 1\right) &&, \text{ if } k = N, \end{aligned} \tag{3}$$
where $d(s, t)$ is the Baseline Forward Discount Factor, y_1 is the Current Coupon Period Year Fraction, and y_N is the Final Coupon Period Year–Fraction.

8. **Baseline Par Curve.** The term “Baseline Par Curve” means the function that assigns to each date t after the Standard Settlement Date T_0 the baseline par yield $p(t)$. It is defined by setting $p(\tau_k) = p_k$ for each maturity date τ_k in the Designated Yield Curve and using linear interpolation for all other dates between τ_1 and τ_M. If a date t is between the Standard Settlement Date and the first maturity date τ_1, then $p(t) = p(\tau_1)$. If a date t is after the Last Yield Curve Maturity Date τ_M, then $p(t) = p(\tau_M)$.

9. **Baseline Spot Curve.** The term “Baseline Spot Curve” means the function that assigns to each date t after the Standard Settlement Date a Baseline Spot Rate $z(T_0, t)$.

   If a date t is within six months of the Standard Settlement Date, then the spot rate is equal to the par yield. In other words,

$$z(T_0, t) = p(t), \text{ if } 0 < y(T_0, t) \leq 0.5. \tag{4}$$

   Otherwise, the Baseline Spot Curve is constructed using the bootstrap procedure and linear interpolation described in Section C below.

10. **Baseline Spot Rate.** The term “Baseline Spot Rate” for a given date t will be denoted $z(T_0, t)$. It is used to compute the Baseline Discount Factor $d(T_0, t)$. The procedure for computing $z(T_0, t)$ is described in Section C.

11. **Black ’76 Formula.** The term “Black ’76 Formula” means the formula for the price of an option on a forward contract developed by Fisher Black.[1]

12. **Bullet Price.** The term “Bullet Price”, denoted P_{bullet}, means the sum of the remaining cashflows of an ECS, discounted using the appropriate OAS–Adjusted Discount Factors. It is given by

$$P_{\text{bullet}} = 100D(T, t_N) + c\left[y_1 D(T, t_1) + \frac{1}{2}\sum_{k=2}^{N-1} D(T, t_k) + y_N D(T, t_N)\right], \tag{5}$$

   where c is the Coupon Rate, y_1 is the Current Coupon Period Year–Fraction, y_N is the Final Coupon Period Year–Fraction, and $t_1, \ldots, t_N$ are the Coupon Payment Dates remaining after the Actual Settlement Date, and $D(T, t_k)$ are OAS–Adjusted Discount Factors

   The Bullet Price can be computed from the Forward Bullet Price $P_{\text{bullet}}(t_1)$ by

$$P_{\text{bullet}} = \begin{cases} D(T, t_1)(cy_1 + P_{\text{bullet}}(t_1)) \text{ if } N > 1 \\ D(T, t_1)(100 + cy_1) \text{ if } N = 1 \end{cases} \tag{6}$$

   where $D(T, t_1)$ is the OAS–Adjusted Discount Factor for the Coupon Payment Date t_1, c is the Coupon Rate, and y_1 is the Current Coupon Period Year–Fraction.

13. **Call Date.** The term “Call Date” means the Coupon Payment Date on which an ECS that has been called must be delivered to the Issuer and on which the Issuer must pay the holder of record the outstanding principal amount. In this Appendix A the Call Date is denoted t_n and is assumed to be after the Actual Settlement Date T.

14. **Call Notification Date.** The term “Call Notification Date” means the date by which the Issuer must notify holders of an ECS that the ECS will be called on the Call Date. Please note that the Call Notification Date is not used in the BMA ECS Formula.

[1] Fisher Black, *The Pricing of Commodity Contracts,* Journal of Financial Economics 3 (1976), 167–179; John C. Hull, *Options, Futures, and Other Derivative Securities*, Prentice Hall, New York, 2000.

15. **Coupon Payment Date.** The term "Coupon Payment Date" means a date on which interest is due and payable on an ECS. The Coupon Payment Dates that remain after the Actual Settlement Date are denoted by $t_1, \ldots, t_N$. The date t_N is the Maturity Date of the ECS.

16. **Coupon Rate.** The term "Coupon Rate", denoted by c, means the annualized coupon rate of an ECS, given as a percentage of Face Amount. The coupon payment for a Regular Coupon Period is therefore $c/2$.

17. **Current Coupon Period.** The term "Current Coupon Period" means the earliest coupon period for which the Coupon Payment Date is after the Actual Settlement Date.

18. **Current Coupon Period Year–Fraction.** The term "Current Coupon Period Year–Fraction", denoted y_1, means the year–fraction of the Current Coupon Period. It is used for computing the coupon payment amount for the current period. If the Current Coupon Period is a Regular Coupon Period, then $y_1 = 0.5$. Otherwise, the Current Coupon Period Year–Fraction should be computed according to the appropriate standard formulas.[2]

19. **Designated Yield Curve.** The term "Designated Yield Curve" has the meaning set forth in the Guidelines and Appendix C. It consists of a discrete set of maturity dates $\tau_1, \ldots, \tau_M$ and corresponding Constant Maturity Yields $p_1, \ldots, p_M$, as defined in Appendix C.

20. **Face Amount.** The term "Face Amount" means the amount of the final principal payment at maturity for one unit of an ECS. The Face Amount is assumed in this Appendix A to be \$100.

21. **Final Coupon Period Year–Fraction.** The term "Final Coupon Period Year–Fraction", denoted y_N, means the year–fraction of the last coupon period of the ECS. If the last coupon period is a Regular Coupon Period, then $y_N = 0.5$. Otherwise, it should be computed using the appropriate formulas.[3]

22. **Forward Bullet Price.** The term "Forward Bullet Price" for a Coupon Payment Date t_k, denoted $P_{\text{bullet}}(t_k)$, means the forward price of an ECS on that date, excluding all cashflows that occur on that date. It is defined recursively as follows:

$$P_{\text{bullet}}(t_k) = \begin{cases} 0 & \text{if } k = N, \\ D(t_{N-1}, t_N)(100 + cy_N) & \text{if } k = N-1, \\ D(t_k, t_{k+1})\left(P_{\text{bullet}}(t_{k+1}) + \frac{c}{2}\right) & \text{if } 1 \le k < N-1, \end{cases} \tag{7}$$

where $D(s, t)$ is the OAS–Adjusted One–Period Forward Discount Factor, c is the Coupon Rate, and y_N is the Final Coupon Period Year–Fraction.

[2] Jan Mayle, *Standard Securities Calculation Methods*, vol. 1, Securities Industry Association, New York, 1993, pages 27–33.

[3] Jan Mayle, *Standard Securities Calculation Methods*, vol. 1, Securities Industry Association, New York, 1993, pages 27–33.

23. **Issue Date.** The term "Issue Date" means the date on which an ECS is issued and interest begins to accrue.

24. **Last Yield Curve Maturity Date.** The term "Last Yield Curve Maturity Date", denoted τ_M, is the last maturity date in the Designated Yield Curve.

25. **Maturity Date.** The term "Maturity Date" means the originally scheduled date on which the principal amount of an ECS becomes due and payable, and ceases to earn interest in accordance with its terms. It is the last Coupon Payment Date and in this Appendix A denoted by t_N.

26. **Odd Coupon Period.** A coupon period that is not exactly six months long. Only the first and last coupon periods of an ECS can be Odd Coupon Periods.

27. **Option–Adjusted Spread (OAS).** The term "Option–Adjusted Spread" ("OAS") means the option adjusted spread in basis points of a particular ECS to the Designated Yield Curve. In this Appendix A it is denoted by δ.

28. **OAS–Adjusted Discount Factor.** The term "OAS–Adjusted Discount Factor" for a Coupon Payment Date t_k is denoted by $D(T, t_k)$ and computed using the OAS–Adjusted One–Period Forward Discount Factors as follows:

$$D(T, t_k) = D(T, t_1) D(t_1, t_2) \cdots D(t_{k-1}, t_k). \tag{8}$$

29. **OAS–Adjusted Forward Par Yield.** The term "OAS–Adjusted Forward Par Yield", denoted F, of the underlying bullet bond as of the Call Date t_n satisfies the equation

$$100 D(T, t_n) = 100 D(T, t_N) + \frac{F}{2} \sum_{k=n+1}^{N} D(T, t_k),$$

where $D(s, t)$ is the OAS–Adjusted Forward Discount Factor. Solving for F, we get

$$F = 200 \frac{D(T, t_n) - D(T, t_N)}{\sum_{k=n+1}^{N} D(T, t_k)}.$$

Equivalently, the OAS–Adjusted Forward Par Yield F can be computed from the Forward Bullet Price $P_{\text{bullet}}(t_n)$ and the Total Coupon Price $C(t_n)$ by

$$F = c \left(1 + \frac{100 - P_{\text{bullet}}(t_n)}{C(t_n)} \right), \tag{9}$$

where c is the Coupon Rate.

30. **OAS–Adjusted One–Period Forward Discount Factor.** The term "OAS–Adjusted One–Period Forward Discount Factor" for the coupon period ending on a Coupon Payment Date t_k is given by

$$\begin{aligned} D(T, t_1) &= \left(1 + \frac{F(T, t_1)}{100}(y_1 - y(t_0, T))\right)^{-1}, \text{ if } k = 1 \\ D(t_{k-1}, t_k) &= \left(1 + \frac{F(t_{k-1}, t_k)}{200}\right)^{-1}, \text{ if } 1 < k < N \\ D(t_{N-1}, t_N) &= \left(1 + \frac{F(t_{N-1}, t_N)}{100} y_N\right)^{-1}, \text{ if } k = N, \end{aligned} \tag{10}$$

where t_0 is the Start of the Current Coupon Period, $F(s, t)$ is the OAS–Adjusted One–Period Forward Rate, y_1 is the Current Coupon Period Year–Fraction, $y(t_0, T)$ is the Accrual Year–Fraction, and y_N is the Final Coupon Period Year–Fraction.

31. **OAS–Adjusted One–Period Forward Rate.** The term "OAS–Adjusted One–Period Forward Rate" for a period starting from a date s to a date t is given by

$$F(s, t) = f(s, t) + \frac{\delta}{100}, \tag{11}$$

where $f(s, t)$ is the Baseline One–Period Forward Rate and δ is the OAS.

32. **Option Price.** The term "Option Price", denoted P_{option}, is computed using the Black '76 Formula as follows.

Let δ be the actual number of days from the Trade Date to the Call Date, and let

$$\sigma = \frac{v_{\text{adjusted}}}{100}\sqrt{\frac{\delta}{365.25}}. \tag{12}$$

The Option Price P_{option} on the Actual Settlement Date is given by

$$P_{\text{option}} = D(T, t_n)C(t_n)\left[N(x_+) - \frac{F}{c}N(x_-)\right], \tag{13}$$

where $D(T, t_n)$ is the OAS–Adjusted Discount Factor for the Call Date t_n, $C(t_n)$ is the Total Coupon Price for the Call Date t_n, F is the OAS–Adjusted Forward Par Yield, c is the Coupon Rate, N is the cumulative normal distribution function, and

$$\begin{aligned} x_+ &= \frac{1}{\sigma}\ln\left(\frac{c}{F}\right) + \frac{\sigma}{2}, \\ x_- &= x_+ - \sigma. \end{aligned}$$

33. **Purchase Price.** The term "Purchase Price", denoted in this Appendix by P, means the full purchase price of an ECS, including the Accrued Interest, assuming a Face Amount of \$100. It is given by

$$P = P_{\text{bullet}} - P_{\text{option}}, \tag{14}$$

where P_{bullet} is the Bullet Price and P_{option} is the Option Price.

34. **Regular Coupon Period.** The term "Regular Coupon Period" means a coupon period that is exactly six months long.

35. **Semiannual Anniversary.** The term "Semiannual Anniversary" of a date s means any date t such that $y(s,t) = 0.5k$, for some positive integer k.

36. **Semiannual Bond Equivalent Yield.** The term "Semiannual Bond Equivalent Yield" means the yield–to–maturity of a coupon-bearing bond, as defined by standard formulas.[4] If the bond has only one coupon period remaining, then the Semiannual Bond Equivalent Yield is a simple yield.

37. **Skew Adjusted Volatility.** The term "Skew Adjusted Volatility" means, with respect to a particular ECS, the volatility that is computed using the relevant Base Volatility, Designated Yield Curve, and particular terms of the security as detailed in Appendix B of these Guidelines. It is used in the computation of the Option Price.

38. **Start of Current Coupon Period.** The term "Start of Current Coupon Period" will be denoted by t_0. If the Actual Settlement Date is before the first Coupon Payment Date of an ECS, then the Start of the Current Coupon Period is the Issue Date. Otherwise, it is the last Coupon Payment Date on or before the Settlement Date.

39. **Standard Settlement Date.** The term "Standard Settlement Date" is defined to be the first business day following the Trade Date that is not an Association recommended holiday. The Standard Settlement Date will be denoted T_0.

40. **Total Coupon Price.** The term "Total Coupon Price" for a Coupon Payment Date t_k, denoted $C(t_k)$, is the OAS–adjusted forward value of the coupon cashflows remaining after that date. It is defined recursively as follows:

$$C(t_k) = \begin{cases} 0 & \text{if } k = N \\ D(t_{N-1}, t_N) c y_N & \text{if } k = N-1 \\ D(t_k, t_{k+1}) \left(\frac{c}{2} + C(t_{k+1})\right) & \text{if } 1 \le k < N-1, \end{cases} \tag{15}$$

where $D(t_k, t_{k+1})$ is the OAS–Adjusted Forward Discount Factor for the period between the Coupon Payment Dates t_k and t_{k+1}, c is the Coupon Rate, and y_N is the Final Coupon Period Year–Fraction.

41. **Trade Date.** The term "Trade Date" is defined to be the date on which the seller and buyer of the European Callable Security agree to the terms of the transaction.

42. **Year–Fractions Between Two Dates.** Given dates $s < t$, define

$$y(s,t) = \frac{\text{Daycount(s, t)}}{360}, \tag{16}$$

where Daycount(s,t) is the number of days between s and t according to the 30/360 daycount convention, computed using the appropriate standard formulas.[5]

[4] Jan Mayle, *Standard Securities Calculation Methods*, vol. 1, Securities Industry Association, New York, 1993, pages 17–23.

[5] Jan Mayle, *Standard Securities Calculation Methods*, vol. 1, Securities Industry Association, New York, 1993,

C. The BMA ECS Formula

1. **Constructing the Baseline Spot and Discount Curves**

   For each date t such that $y(T_0, t) \leq 0.5$,

   $$z(T_0, t) = p(t),$$

   where $p(t)$ is the Baseline Par Curve.

   Let $s_1, \ldots, s_{60}$ denote the Semiannual Anniversary dates of the Standard Settlement Date out to 30 years. Since the year–fraction $y(T_0, s_1) = 0.5$, the Baseline Spot Rate for the date s_1 is

   $$z(T_0, s_1) = p(s_1),$$

   The corresponding discount factor $d(T_0, s_1)$ is obtained using (1).

   Now assume that Baseline Discount Factors $d(T_0, s_1), \ldots, d(T_0, s_k)$ have already been solved for. The Baseline Discount Factor $d(T_0, s_{k+1})$ is given by the following bootstrap formula:

   $$d(T_0, s_{k+1}) = \frac{1 - \frac{p(s_{k+1})}{200} \sum_{j=1}^{k} d(T_0, s_j)}{1 + \frac{p(s_{k+1})}{200}}. \tag{17}$$

   This recursively defines the Baseline Discount Factors $d(T_0, s_1), \ldots, d(T_0, s_{60})$. The corresponding Baseline Spot Rates $z(T_0, s_1), \ldots, z(T_0, s_{60})$ are given by the formula

   $$z(T_0, s_k) = 200 \left(\frac{1}{[d(T_0, s_k)]^{1/(2y(T_0, s_k))}} - 1 \right) \tag{18}$$

   where $y(T_0, t)$ is the Year–Fractions Between Two Dates T_0 and t.

   For all other dates t, the Baseline Spot Curve is defined using linear interpolation.

   The Baseline Discount Factor Curve is defined using equation (1) for all times t after the Standard Settlement Date T_0.

2. **Computation of the Purchase Price from the OAS**

   2.1 **Computation of the Bullet Price.** The recommended procedure for computing the Bullet Price P_{bullet} is the following:

   2.1.1 Compute the OAS–Adjusted One–Period Forward Discount Factor for each remaining coupon period using (10).

   2.1.2 Compute the Forward Bullet Price for each remaining Coupon Payment Date using (7).

   2.1.3 Compute the Bullet Price using (6).

   2.2 **Computation of the Option Price.** The recommended procedure for computing the Option Price P_{option} is the following:

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2.2.1 Compute the Total Coupon Price for Call Date using (15).

2.2.2 Compute the Forward Bullet Price for the Call Date using (7).

2.2.3 Compute the OAS–Adjusted Forward Par Yield using (9).

2.2.4 Compute the Option Price using (13).

2.3 **Computation of the Purchase Price** The Purchase Price is computed from the Bullet Price and the Option Price using (14).

3. **Computation of OAS from the Purchase Price**

   If the Purchase Price is known, the corresponding OAS can be computed using iterative methods such as the Newton–Raphson method or the bisection method.